`r\_{CM} = 1/M sum(i=0)^n m\_{i}r\_{i}`

`r\_{CM}` is the position of center of mass in a discrete mass distribution

Two particle system of mass `m\_{1}` and `m\_{2}` separated by a distance r\

`m\_{1}r\_{1} =m\_{2}r\_{2}`

`r\_{1}, r\_{2}` are distances from center of mass to respective masses

distance of com to mass is inversely proportional to mass

`r propto 1/m `

COM lies mid-way of two equal masses

COM lies closer to larger mass if the masses are unequal

Center of mass of continues mass distribution

`x\_{CM} = (intxdm)/int dm`

`y\_{CM} = (intydm)/int dm`

`z\_{CM} = (intzdm)/int dm`

`r\_{CM} = 1/Mint rdm`

center of mass of uniform rod

`x\_{cm} =L/2` L is the length

`y\_{cm} = 0 z\_[cm} = 0`

Coordinates of COM (`L/2`, 0, 0)

Center of mass of semicircular ring

`y\_{cm} = (2R)/pi`

center of mass of semicircular disc

`y\_{cm} = (4R)/(3pi)`

center of mass of solid hemisphere

`y\_{cm} = (3R)/8`

center of mass of a hollow hemisphere

`y\_{cm} = R/2`

center of mass of solid cone

`y\_{cm} = (3H)/4`

center of mass of hollow cone

`y\_{cm} = (2H)/3`

center of mass of rectangular plate

`x\_{cm} = b/2 y\_{cm}=L/2`

center of mass of triangular plate is at centroid

`y\_{cm} = h/3` h =height

For 1D structures like rod, ring derivation we use mass per unit length

For 2D structures like rectangle, square we use mass per unit area

For 3D structures like cube, sphere we use mass per unit volume

if some mass or area removed from a rigid body `r\_{cm}` of remaining body is obtained by

`r\_{cm} = (m\_{1}r\_{1} – m\_{2}r\_{2})/(m\_{1} – m\_{2})`

or

`r\_{cm} = (A\_{1}r\_{1} – A\_{2}r\_{2})/(A\_{1} – A\_{2})`

If r about x, y, z is asked just substitute x, y, z in r

Velocity of COM

`V\_{cm} = (m\_{1}v\_{1} + m\_{2}v\_{2} + …. + m\_{n}v\_{n})/M`

M = total mass = `m\_{1} + m\_{2}…..`

Acceleration of COM

`a\_{cm} = (m\_{1}a\_{1} + m\_{2}a\_{2} + …. + m\_{n}a\_{n})/M = text(net external force divided by M)`

`F\_{ext} = ma\_{cm}`

`F\_{ext} = 0`, `a\_{cm} = 0`, `v\_{cm}` = constant so momentum constant

Momentum conservation

`F\_{ext} = (dP)/(dt)`

If `F\_{ext} = 0` then `(dP)/(dt) = 0`

That is P = constant

`P\_{1} + P\_{2} +……. P\_{n} = text(constant)`

Eg: ` m\_{1}v\_{1} + m\_{2}v\_{2} = 0`

Application : when a projectile explodes in air to different parts, there no change in the path of center of mass because during explosion no external force (except gravity) act on COM

`K.E = P^2/(2m)` or `P = sqrt(2K.Em)`

`text(Percentage change) = (text(final – initial))/text(initial) \*100`

Variable mass

Eg: Rocket, mass reduction

In such problems use thrust force

`F\_{t} = v\_{rel}(pm(dm)/(dt))`

`v\_{rel}` is the velocity of mass gained or lost relative to main mass

`(dm)/(dt) = `rate at which mass increasing or decreasing

Linear impulse (J)

`J = Ft = DeltaP`

Impulse is area under F-t graph `= DeltaP`

continue

`(dL)/(dt) = r xx F = tau`